Optimal Tilt Angle and Orientation for Solar Collectors in Iran

Farnaz Safdarian and Mohammad Esmaeil Nazari

Abstract—The variation of tilt angle changes the amount of solar radiation that reaches to the surface of the collector. Hence, tilt angle is an important factor that affects the performance of a solar collector. In this study, a mathematical model is proposed for estimating the solar radiation on a tilted surface, which determines the optimum tilt angle of solar collector and its orientation (surface azimuth angle) in a specific period of time during a clear day, in some big cities of Iran. The optimum angles are calculated using GSA for the values of which the radiation on the collector surface is at the maximum level for different circumstances of the environment. The results reveal that setting the tilt angle and surface azimuth angle are necessary in order to reach to the maximum radiation in clear days when $\cos \theta = 1$ and angle of incidence would be zero.

Keywords—Azimuthal angle, optimization, tilt angle, solar collector.

I. INTRODUCTION

Solar energy, especially in tropical and subtropical regions is one of the most promising renewable energy sources. Along with other forms of renewable energy sources (i.e. winds, geothermal, sea waves and biomass), it has a great potential for a wide variety of applications because of its abundance and accessibility. Solar systems, like any other system, need to be operated with the maximum possible performance. This can be achieved by proper design, construction, installation, and orientation. Therefore, the performance of a solar collector is highly influenced by its orientation (regarding the Equator) and its tilt angle (regarding the ground). This is due to the fact that both the orientation (surface azimuth angle) and tilt angle affect the solar radiation that reaches to the surface of the collector [1]. Several interesting articles [1–14] have been devoted to this problem. In [1] and [2], the results reveal that changing the tilt angle 12 times in a year maintains approximately the total amount of solar radiation near the maximum value that is found by changing the tilt angle daily to its optimum value. In [3] and [4], the annual optimal angles for various cities in Taiwan and Canada are determined, respectively. As the main contributions of this study, Gravitational Search Algorithm (GSA) is used to find maximum incidence angle. Also, a study in Iran because of its high potential to produce solar energy is necessary. The objective of this study is to find the optimal tilt angle and surface azimuth angle of solar collector on a clear day in major cities of Iran.

The system description is shown in section II. Section III formulates the optimization problem and in section IV, the optimization algorithm is discussed. The simulation results are presented in section V and finally, in section VI, conclusion and recommendations are presented.

II. DESCRIPTION OF THE SYSTEM

Fig. 1 shows the sun and solar collector angles that are used for the optimization [1]. As it is shown in Fig. 1, the radiation angle of sun that is received by tilted collector is called solar zenith angle ($\theta$). For solar collector, the two important angles, which should be adjusted in order to achieve the maximum energy from the sun, are tilt angle ($\beta$) and azimuth angle ($\gamma$). Furthermore, careful attention should be paid to the incidence angle ($\delta$), the angle between the sun's rays and the normal vector to the tilted surface. If $\theta = 0$, the surface receives the maximum radiation beam from the sun.

![Fig. 1. Sun and collector angles.](#)
III. PROBLEM FORMULATION

The hourly total radiation on a tilted surface (\( I_T \)) is [4]

\[ I_T = I_s R_b + I_d F_{v,s} + I \rho_s F_{v,g} \]  

(1)

Moreover, the total radiation on a tilted surface during a day (\( H_T \)) and the annual total radiation (\( E \)) are obtained from (2) and (3), respectively [4]:

\[ H_T = \sum_{n} I_T \]  

(2)

\[ E = \sum_{n} \sum_{m} I_r \]  

(3)

where [4],

\[ I_o = I - I_d \]  

(4)

\[ I_d = I(1-0.09K_r) \]  

(5)

\[ I_d = I(0.95-0.16K_r+4.3(3K_r)^3) \]  

\[ 0.22 \leq K_r < 0.8 \]  

(6)

\[ \cos \theta = \sin \phi \sin \delta \cos \beta + \cos \phi \cos \delta \cos \phi \cos \beta \]  

\[ -\sin \delta \cos \phi \sin \beta \cos \gamma \]  

\[ + \cos \gamma \cos \phi \cos \sin \phi \sin \beta \]  

\[ + \cos \delta \sin \beta \sin \gamma \sin \omega \]  

\[ \cos \theta_\gamma = \cos \phi \cos \delta \cos \phi \cos \beta \]  

(7)

\[ I = \frac{12 \times \sin \beta \sqrt{F_{v,s}}}{\pi} \]  

(8)

\[ I = \frac{12 \times \sin \beta \sqrt{F_{v,g}}}{\pi} \]  

(9)

\[ I_o = \frac{12 \times 3600}{\pi} G_\infty \]  

\[ \left( \cos \phi \cos \delta \sin \omega - \cos \omega \right) + \pi \left( \frac{\omega - \omega_0}{180} \right) \sin \phi \sin \delta \]  

(10)

\[ \omega_0 = 15^\circ v, 12^\prime 00' \]  

(11)

\[ \omega_1 = 15^\circ v, 11^\prime 00' \]  

(12)

\[ \omega = \frac{\omega_0 + \omega_1}{2} \]  

(13)

\[ F_{v,s} = \frac{1+\cos \beta}{2} \]  

(14)

\[ F_{v,g} = \frac{1-\cos \beta}{2} \]  

(15)

\[ \delta = \frac{23.45 \sin 360(284+n)}{365} \]  

(16)

\[ G_\infty = G_\infty \left( 1 + 0.033 \cos \frac{360n}{365} \right) \]  

(17)

Besides, the constant parameters are:

\[ G_\infty = 1367 \text{ W m}^{-2} \]  

(18)

\[ \rho_s = 0.6 \text{ if } n < 60 \text{ (winter)} \]  

(19)

\[ \rho_s = 0.2 \text{ if } n > 59 \text{ (other seasons)} \]  

(20)

IV. SOLUTION METHODOLOGY

The objective of this study is to find the optimal tilt angle and surface azimuth angle of the solar collector on a clear day in major cities of Iran.

In clear days, \( K_r \approx 0.8 \) and \( \rho_s = 0.2 \). Therefore, with a good approximation, it could be considered that \( I_d F_{v,s} \) and \( I \rho_s F_{v,g} \) are negligible, compared to \( I_o \), so:

\[ I_T = I_o R_b \]  

(21)

\( I_o \) is a function of latitude and declination while the angles of the collector are negligible. Hence, the maximum \( I_T \) corresponds to the maximum \( R_b \). In order to maximize \( R_b \), \( \theta \) should be set to zero. It must be noted that \( \theta \) is a function of latitude and declination and remains constant when the collector is rotating.

The objective function of this study in order to find the optimal tilt angle and the surface azimuth angle is shown in (22).

\[ \max \{ \cos \theta \} = 1 = f(\beta, \gamma) \]  

(22)

subject to the following constraints

\[ 0 \leq \beta \leq 90 \]  

(23)

\[ -180 \leq \gamma \leq 180 \]  

(24)

\[ h_s \leq h_\beta \leq h_a \]  

(25)

\[ h_s \leq h_\gamma \leq h_a \]  

(26)

Exact optimization algorithms are not able to provide an appropriate solution for solving optimization problems with a high-dimensional search space. In these problems, the search space grows exponentially with the problem size; therefore the exhaustive search is not practical. Also, classical approximate optimization methods make several assumptions to solve the problems. Sometimes, the validation of these assumptions is difficult in each problem. However, metaheuristic algorithms are robust and can adapt solutions with changing conditions and environment; they can be applied in solving complex multimodal problems; and they may incorporate mechanisms to avoid getting trapped in local optima. Furthermore, these algorithms are able to find promising regions in a reasonable time due to exploration and exploitation ability. Hence, metaheuristic algorithms, which make few or no assumptions about a problem and can search very huge spaces of candidate solutions, have been extensively developed to solve optimization problems these days. Among these algorithms, population-based metaheuristic algorithms are proper for global searches due to global exploration and local exploitation ability. [19]

In this study, Gravitational Search Algorithm (GSA) is used to find the optimal \( \beta \) and \( \gamma \). This algorithm is capable to find the optimum \( \beta \) and \( \gamma \) in every hour of every day, and in every latitude.
GSA is based on the law of gravity and mass interactions. Each mass (agent) has four specifications including: position, inertial mass, active gravitational mass, and passive gravitational mass. Every position of the mass corresponds to one solution of the problem, and gravitational and inertial masses are determined to use a fitness function. In fact, the GSA is navigated by properly adjusting masses. For this reason, the masses obey the Newtonian laws of gravitation and motion. According to the law of gravity, each mass attracts other masses. The gravitational force between two particles is directly proportional to the product of their masses and inversely proportional to the distance between them, \( R \). We used \( R \) instead of \( R^2 \), because the experiment proves that \( R \) provides better results than \( R^2 \). Masses must be attracted by the heaviest one which presents an optimum solution in the search space.

Considering a system with \( N \) agents, the position of the \( i \)th agent is defined by (27):

\[
X_i = (X_i^1, ..., X_i^d, ..., X_i^n) \quad \text{for} \quad i = 1, 2, ..., N \tag{27}
\]

In this problem \( X \) is the same as PV area that is shown by \( A_{PV} \) provided that costs are minimized. The force acting on mass \( i \) from mass \( j \) is defined as (28):

\[
F_{ij}^d(t) = g(t)\frac{M_{pi} \times M_{mj}}{R_{ij}(t) + \varepsilon} (X_j^d(t) - X_i^d(t)) \tag{28}
\]

\( R_{ij}(t) \) is the Euclidian distance between two agents \( i \) and \( j \):

\[
R_{ij}(t) = \|X_j(t), X_i(t)\|_2 \tag{29}
\]

To give a stochastic characteristic to the algorithm, in order to search the possible space more efficiently, the total force that acts on agent \( i \) in dimension \( d \), \( F_{i}^d \) would be randomly weighted sum of \( d \)th components of the exerted forces from other agents. To improve the performance of GSA by controlling exploration and exploitation, it is assumed that only the \( K_{best} \) agents will attract the others. \( K_{best} \) is a function of time, with the initial value \( K_0 \) at the beginning and decreases with time. At the beginning, all agents apply the force, but as time passes, \( K_{best} \) is decreased linearly and at the end, only 2% of the agents apply force to the others. Thus, \( K_{best} \) is the set of first \( K \) agents with the best fitness value and the biggest mass.

\[
F_{i}^d(t) = \sum_{j \in K_{best}, j \neq i} \text{rand}_j F_{ij}^d(t) \tag{30}
\]

where \( \text{rand}_j \) is a random number in the interval \([0,1]\). According to the law of motion, the acceleration of the agent \( i \) at time \( t \), and in direction \( d \)th (\( a_i^d \)), is as (31):

\[
a_i^d(t) = \frac{F_i^d(t)}{M_i^t(t)} \tag{31}
\]

The next position and velocity could be calculated as:

\[
v_i^d(t + 1) = \text{rand}_i \times v_i^d(t) + a_i^d(t) \tag{32}
\]

\[
x_i^d(t + 1) = x_i^d(t) + v_i^d(t + 1) \tag{33}
\]

\( \text{rand}_i \) is used to give a randomized characteristic to the search.

The gravitational factor \( g \) is initialized at the beginning and will be reduced with time to control the search accuracy. In other words, \( g \) is an exponential function of the initial value and time. Gravitational and inertia masses are simply calculated by the fitness evaluation. A heavier mass is a more efficient agent. Assuming the equality of masses, they are calculated using the map of fitness. The gravitational and inertial masses are updated in every iteration by (34-36):

\[
M_{ai} = M_{pi} = M_{ii} = M_i, \quad i = 1, 2, ..., N \tag{34}
\]

\[
m_j(t) = \frac{\text{fit}_j(t) - \text{worst}(t)}{\text{best}(t) - \text{worst}(t)} \tag{35}
\]

\[
M_j(t) = \frac{m_j(t)}{\sum_{j=1}^{N} m_j(t)} \tag{36}
\]

The optimal value of the former iteration is saved as \( \text{fit} \). In this problem \( \text{fit} \) is the cost function. For minimization problems, (37) and (38) are used: [18]

\[
\text{best}(t) = \min \{\text{fit}_j(t)\} \quad j \in \{1, ..., N\} \tag{37}
\]

\[
\text{worst}(t) = \max \{\text{fit}_j(t)\} \quad j \in \{1, ..., N\} \tag{38}
\]
V. SIMULATION RESULTS

Five major regions of Iran, which are Tehran ($\phi = 35.7^\circ$), Isfahan ($\phi = 32^\circ$), Mashhad ($\phi = 36.2^\circ$), Tabriz ($\phi = 38^\circ$), and Shiraz ($\phi = 29.4^\circ$), are studied in order to find the optimal angles.

To find sunset and sunrise hour angles of these cities, the following equations are used.

$$ h_s = \cos^{-1}(-\tan \phi \tan \delta) $$
$$ h_r = -h_s $$
$$ H_s = \frac{1}{15} h_s $$

and daylight hours is $2H_s$.

The simulation results are presented for sample days of winter ($n = 1$), spring ($n = 91$), summer ($n = 182$), and fall ($n = 274$).

Tables I and II show the optimum tilt angle and the surface azimuth angle for five cities of Iran.

To verify the accuracy of the optimum angles, in Table III, the objective function value ($\cos \theta$) is depicted. It is shown that the maximum $\cos \theta$ and therefore the maximum radiation is reached on clear days. Table IV shows the results from the objective function for five major cities of Iran on sample days in different seasons and hours.

### Table II
<table>
<thead>
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<th>City</th>
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<td>90</td>
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<td>Tabriz</td>
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<td>80</td>
<td>70</td>
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<td>43</td>
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<tr>
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<td>-54</td>
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<td>-31</td>
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<td>0</td>
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<td>63</td>
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<tr>
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### TABLE IV
Results from the Objective Function for Five Major Cities of Iran on Sample Days in Different Seasons and Hours

<table>
<thead>
<tr>
<th>City</th>
<th>$n$</th>
<th>Objective function ($\cos \theta$)</th>
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<td>91</td>
<td>0.8</td>
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<tr>
<td>274</td>
<td>1.0</td>
<td>1.0</td>
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<tr>
<td>Isfahan</td>
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<td>1.0</td>
</tr>
<tr>
<td>32.°</td>
<td>91</td>
<td>0.8</td>
</tr>
<tr>
<td>274</td>
<td>1.0</td>
<td>1.0</td>
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<td>Shiraz</td>
<td>1</td>
<td>1.0</td>
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<tr>
<td>29.4°</td>
<td>91</td>
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<td>36.2°</td>
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<tr>
<td>274</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figs. 2-6 show the variation of radiation with hourly optimum tilt angle, $\beta_{opt}$ for different latitudes in 12 p.m. that leads to the maximum energy.

It can be seen that the most radiation happens in $\gamma=0$ and $\beta=55$.

Fig. 2. The radiation versus tilt angle in 12 p.m. day 182 for N=35.7 in Tehran.

It can be seen that the most radiation happens in $\gamma=0$ and $\beta=26$.

Fig. 3. The radiation versus tilt angle in 12 p.m. day 1 for N=32 in Isfahan.

Fig. 4. The radiation versus tilt angle in 12 p.m. day 91 for N=29.4 in Shiraz.

Fig. 5. The radiation versus tilt angle in 12 p.m. day 274 for N=36.2 in Mashhad.
It can be seen that the most radiation happens in $\gamma=0$ and $\beta=41$.

In this case, the most radiation happens in $\gamma=0$ and $\beta=34$.

VI. CONCLUSION

In this study, a mathematical model is used for estimating the solar radiation on a tilted surface, in order to determine the optimum tilt angle and orientation (surface azimuth angle) for the solar collector in the main cities of Iran, on a clear day, and on a specific period of time. It is obvious that in order to reach to the maximum radiation on clear days, the maximum $\cos \theta$ is needed. To do this, the optimum tilt angle and surface azimuth angle are calculated. It is recommended to apply this algorithm for various types of days (such as clear, cloudy, rainy) and in various places.

APPENDIX

TABLE I

<table>
<thead>
<tr>
<th>NOMENCLATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ Total annual radiation</td>
</tr>
<tr>
<td>$F_{c-g}$ Radiation view factor between ground and collector</td>
</tr>
<tr>
<td>$F_{c-s}$ Radiation view factor between the sky and the collector</td>
</tr>
<tr>
<td>$G_{on}$ Extraterrestrial normal radiation</td>
</tr>
<tr>
<td>$G_{sc}$ Solar constant</td>
</tr>
<tr>
<td>$H_T$ The total radiation on a tilted surface during a day</td>
</tr>
<tr>
<td>$h_1$ Sunrises hour angle</td>
</tr>
<tr>
<td>$h_2$ Sunset hour angle</td>
</tr>
<tr>
<td>$H_{st}$ Sunset hour</td>
</tr>
<tr>
<td>$I$ Total radiation for an hour on a horizontal surface</td>
</tr>
<tr>
<td>$I_b$ Total beam radiation for an hour on a horizontal surface</td>
</tr>
<tr>
<td>$I_d$ Total diffuse radiation for an hour on a horizontal surface</td>
</tr>
<tr>
<td>$I_o$ The total solar radiation incident on an extraterrestrial horizontal surface during an hour</td>
</tr>
<tr>
<td>$I_T$ Total radiation for an hour on a tilted surface</td>
</tr>
<tr>
<td>$m$ Number of month</td>
</tr>
<tr>
<td>$N$ Number of daylight hours</td>
</tr>
<tr>
<td>$n$ Day number of year</td>
</tr>
<tr>
<td>$n_{month}$ The number of month’s day</td>
</tr>
<tr>
<td>$R_g$ Geometric factor</td>
</tr>
<tr>
<td>$t$ Time period index</td>
</tr>
<tr>
<td>$t_1, t_2$ First and end hour of a time period</td>
</tr>
<tr>
<td>$\beta$ Surface tilt angle</td>
</tr>
<tr>
<td>$\gamma$ Surface azimuth angle</td>
</tr>
<tr>
<td>$\delta$ Declination</td>
</tr>
<tr>
<td>$\theta$ Incidence angle</td>
</tr>
<tr>
<td>$\theta_z$ Solar zenith angle</td>
</tr>
<tr>
<td>$\rho_g$ Ground reflectance</td>
</tr>
<tr>
<td>$\phi$ Latitude</td>
</tr>
<tr>
<td>$\omega$ Hour angle</td>
</tr>
<tr>
<td>$\omega_1, \omega_2$ First and end hour angles of a time period</td>
</tr>
</tbody>
</table>

REFERENCES

AUTHORS’ INFORMATION

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